

# ON THE LS-CATEGORY AND TOPOLOGICAL COMPLEXITY OF PROJECTIVE PRODUCT SPACES

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**ABSTRACT.** We determine the Lusternik-Schnirelmann category of the projective product spaces introduced by D. Davis. We also obtain an upper bound for the topological complexity of these spaces, which improves the estimate given by J. González, M. Grant, E. Torres-Giese, and M. Xicoténcatl. We conclude an exact value of the topological complexity for some cases. Thus, we finalize the estimating problem on the topological complexity of these spaces.

In this note, we first compute the LS-category of  $P_{\bar{n}}$ :

**Theorem 0.1.**  $\text{cat}(P_{\bar{n}}) = \text{cat}(P^{n_1}) + r - 1 = n_1 + r - 1$ .

For the topological complexity, we establish:

**Theorem 0.2.**  $\text{TC}(P_{\bar{n}}) \leq \text{TC}(P^{n_1}) + \sum_{q=2}^r \text{TC}(S^{n_q})$ .

**Corollary 0.3.** *If  $\text{zcl}_{\mathbb{Z}_2}(P^{n_1}) = \text{TC}(P^{n_1})$  and  $n_q$  is odd for  $q > 1$  then  $\text{TC}(P_{\bar{n}}) = \text{TC}(P^{n_1}) + \sum_{q=2}^r \text{TC}(S^{n_q}) = \text{TC}(P^{n_1}) + r - 1$ .*

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